Foresighted Network Formation and Network Games

Yangbo Song   Mihaela van der Schaar

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Abstract

A network determines how individuals may interact with each other as well as how agents may learn about each other's behavior. However, the theoretical literature often regards network formation, network games and network monitoring as isolated problems. In this paper, we propose a framework that unifies these important functions of a network. An agent in our model repeatedly chooses whom to link with, and in the network that thus forms, what actions to take in every network-dependent game. We find strong interdependency between the equilibrium networks and the equilibrium actions, and that endogenously formed networks with associated action profiles often yield a higher social welfare than exogenously prescribed ones. In addition, we emphasize the importance of an informative public monitoring structure, and show that without such monitoring cooperation may fail in a sparse network even if agents are very patient.

Keywords: Network Formation, Network Games, Foresight, Cooperation, Monitoring

JEL Classification: A14, C72, C73, D83, D85

1 Introduction

Networks have an ever-growing influence on various social and economic interactions, including trade, public good provision, friendship, information sharing, etc. In a network, agents are linked via relation such as business partnership, social activities and user protocol, and an agent's well being is affected by not only *how* these links are structured – which other agents constitute her "neighborhood" – but also *what* actions are taken by the other party of each relevant link. For example, in a business network in the wine industry consisting of wineries, distributors and sales brokers, the profit of a participant depends on both the group of other businesses it establishes partnership with, as well as the terms of each contract it signs. A large and growing empirical literature (e.g. Karlan et al.[26], Jackson et al.[21], Ambrus et al.[2]) emphasizes the importance of networks on individual behavior and social outcomes. From the theoretical perspective, a network serves two purposes in each of these strategic environments. On one hand, it is a platform for interaction since being in a particular neighborhood is a prerequisite for interacting with each agent in this neighborhood. On the other hand, it is a channel for information since detailed information of both links and actions in unfamiliar neighborhoods is often limited, if not unavailable at all. In addition, the structure of a network itself is often the result of strategic decisions by the agents, who would weigh the benefit and cost of every feasible link before forming one. In this paper, we develop a model to unify these crucial features of a network. We would like to understand how the network topology as well as agents' strategic behavior are shaped by the interplay between network formation and network games. Also, we would like to evaluate the implication of different environments – different networks, different games or different monitoring structures – on social welfare.

The existing approaches on analyzing networks and games played on a network are often constrained by focusing on only one aspect of the whole environment. Typical models include network formation without associated action space (e.g. Jackson and Wolinsky[23], Bala and Goyal[3], Watts[34]), games played on a fixed network (e.g. Galeotti et al.[15], Ballester et al.[4], Bramoulle and Kranton[5]), networks as pure information exchange devices (e.g. Ali and Miller[1], Wolitzky[35]), etc. This paper connects the ideas behind these studies. From the perspective of network formation, this paper is the first to introduce two important features of a network: determining the underlying games agents play and shaping the information agents receive. From the perspective of network games, this paper contributes to the literature by making network formation part of the agents' strategic choices and contrasting the results to those with exogenous network formation.

In our model, a finite number of agents make strategic decisions on a discrete and infinite time line. In each period there are two stages: network formation and interaction. In the first stage, agents form or change the current network, where links are established by bilateral consent and severed by unilateral intent. The formation or maintenance of a link is costly while the severance of one is not. In the second stage, a game is played on every complete sub-network, and an agent's payoff in the current period is the sum of payoffs from each game in the second stage less her cost in the first stage. The agents are foresighted and discount their future payoffs by a given discount factor. In terms of information, an agent knows every action of every player in every game she plays, but has no knowledge of actions taken in any other game. This implies that she knows every complete sub-network involving herself. The rest of her knowledge on the network is given by a monitoring structure, which is a function of the network formation history that produces a public signal. The monitoring structure may represent various degrees of information, ranging from full knowledge (perfectly revealing the network formation history) to no additional knowledge (independent from the network formation history).

Before introducing the formal results, we would like to emphasize two interesting features of this framework. First, the incentive to form or maintain a certain network depends on the games to be played. For instance, consider a simple environment with three agents and consider the game played on the triangle network (full clique). If this game may yield a significantly higher payoff to every agent than in every other game, the agents have strong incentives to maintain the corresponding network over time. If on the contrary this game can only result in a largely negative payoff for every agent, the network can never be sustained. Second, the level of coordination in games does not only rely on the discount factor as in the classical repeated game literature, but is greatly affected by the network topology as well. For example, consider a series of prisoners' dilemma played on a wheel network (a circle of links). Each game is played by a pair of linked agents. Suppose that the monitoring structure is very uninformative about the formation history, i.e. it is unlikely for an agent to know about link formation or severance outside the set of complete sub-networks she is in. Then any deviation from a given action profile will only trigger a punishment against the deviator after a long time, since any reaction can only affect one new pair of agents at a time on a wheel network. As a result, when the wheel is sufficiently large, full cooperation is impossible in any equilibrium.

Following the ideas behind the above examples, our main results are three-fold. We focus on equilibria that feature long-run persistence of a network and an associated action profile, and discuss their properties and welfare implications. First, we prove a

Convergence Theorem characterizing the set of network and action combinations that can be supported in equilibrium when agents are sufficiently patient. This result offers a novel and more realistic view on networks and network games than the classical Folk Theorem, in the sense that it highlights how the network topology affects the sustainability of an action profile in a given game. For instance, even if an agent receives a negative payoff in one of the games she plays, she may just put up with this loss as long as it can be compensated from positive payoffs in other games; the network determines how she ends up playing all these games and how she gets punished by other agents if she does not follow a prescribed action in each game. The range of action profiles that can be sustained, even on the same network, depends greatly on the monitoring structure. Second, we discuss welfare consequences by connecting network formation and network games. In particular, we show that in generic cases our model with strategic network formation produces a higher maximum social welfare than a model with exogenous network formation. This result suggests that equilibrium outcomes get closer to social efficiency in a more tolerant environment where individuals may establish relationship freely, rather than a controlled environment that heavily regulates how individuals are inter-connected. Third, we generalize the second example above to argue that when the monitoring structure is not informative, cooperation can never be sustained in a network that is too sparse. We believe that this result opens another dimension – aside from the discount factor – for studying cooperative behavior among selfish and strategic agents. It emphasizes that how close or how distant agents are when making their cooperation decisions must be taken into account.

The remainder of the paper is organized as follows: Section 2 reviews related literature. Section 3 introduces the model. Section 4 presents the main results. Section 5 concludes the paper.

2 Literature Review

The theoretical literature related to network formation and games played on networks can be divided into three main caterogies.

First, there is a large and growing literature on modeling network formation as a result of strategic choice. In general, a link between two agents is viewed as access to valuable resources possessed by one another, and hence the network structure fully determines the payoff of each agent. This literature starts with static models with homogeneous agents by Jackson and Wolinsky[23] and Bala and Goyal[3], which are then developed to include additional features such as agent heterogeneity and endogenous resource production (see e.g. Galeotti[13], Galeotti et al.[16], Haller and Sarangi[20], Galeotti and Goyal[14]). The main message from these works is that particular network topologies, such as the star and the clique, can be identified as strongly efficient and/or predicted to emerge in the unique equilibrium.

An important strand of literature that stems from Jackson and Wolinsky[23]'s framework is its sequential or dynamic version with myopic agents, notable works including Johnson and Gilles[24], Deroian[8], Watts[34], Jackson and Watts[22], and Song and van der Schaar[30]. There also has been several approaches on network formation with foresighted agents, for instance Dutta et al.[9] and Song and van der Schaar2[29]. In these models, the networks predicted by static frameworks are usually *not* persistent over time, or there are many other network topologies that can be sustained in equilibrium. In the current paper, our model is also a dynamic one but is different from the above cited ones in a fundamental aspect: apart from network formation, we introduce an associated action space. As a result, the network topology is no longer merely a set of pairwise access, but a guide of what games are played by which agents. As we shall see in the analysis, this is not only a more realistic setting – in many applications, individuals or firms interact in a much more complex way than simply connect to one another and collect fixed payoffs – but the actions that may be undertaken and their consequences in even one single game among many can have a significant impact on network formation.

Second, this paper relates to the theoretical study of games played on a fixed network (Galeotti and Vega-Redondo[17], Ballester et al.[4], Sundararajan[32], Galeotti et al.[15]), with applications such as consumers networks (Galeotti[12]), experimentation and innovation (Bramoulle and Kranton[5]) and R&D networks (Goyal and Moraga-Gonzalez[19]). These works typically focus on the relation between network topology and equilibrium behavior in games, and a number of them also characterize the network topologies that maximize social welfare. Our paper shares the emphasis of networks' impact on sustainable actions such as cooperation behavior in a generalized prisoners' dilemma, and we also derive from our model conditions under which a Pareto improvement may be sustained in equilibrium. Most importantly, in this paper we ask another fundamental question: how does the underlying network topology comes into being? Unlike the literature on network games, we believe that forming a network and playing games on a network can be better understood as a unified process of strategic interaction, instead of the former being governed by an exogenous stochastic rule and only the latter being strategic. This view is supported by the applications of social network and trade network, in which the conduct of individuals or firms after relationships are built affect decisions on whether to form these relationships in the first place.

Third, our analysis the monitoring structure's effect on sustainable networks and actions is inspired by a literature on community monitoring in repeated games. Renowned early papers such as Kandori[25] and Ellison[10] present Folk Theorems which show that cooperation can be enforced even if agents are not perfectly informed about the whole action history. The key argument is that when every possible deviation triggers a "contagion" of punishment that will soon spread to the whole community, deviation is never optimal even when the deviator may not be identified. This idea is developed by Takahashi[33], Deb[7] and Sugaya and Takahashi[31], but their studies have not introduced networks as an underlying structure for monitoring. In two recent papers, Ali and Miller[1] and Wolitzky[35], network topologies are explicitly imposed as a monitoring device: an agent only observes the actions taken by her immediate neighbors. Part of our monitoring structure resembles this approach in the sense that agents only have local observation of actions, i.e. they only observe actions taken in the complete sub-networks that involve themselves. However, since network formation is also endogenous, our monitoring structure includes (possibly imperfect) monitoring on the network formation history as well. How precisely this history is revealed to the agents affects the sustainable network and action profile significantly. Our result is in stark contrast to much of the literature: we show that in some networks cooperation can *never* be sustained even with contagion strategy.

Apart from the theoretical trend of thought, an active empirical literature studies the impact of network structure on sustaining cooperation in various markets (Karlan et al.[26], Jackson et al.[21], Ambrus et al.[2]) and topological properties of real networks in market data and experiments (Falk and Kosfeld[11], Corbae and Duffy[6], Goeree et al.[18], Mele[28], Leung[27]). Two important messages are conveyed in these works. First, an individual's position in a network – how central she is, whether she is the sole connector between two groups, etc. – largely determines the cooperation level she undertakes. Second, equilibrium networks predicted by much of the theoretical literature on network formation only account for a very small fraction of real networks. Our results are consistent with these qualitative assertions, but we will leave the quantitative development of our model for future research.

3 Model

3.1 Network Topology

Consider a group of agents $I = \{1, 2, \cdots, N\}$. A *network* is denoted by $\mathbf{g} \subset \{ij : i, j \in I, i \neq j\}$. ij is called a *link* between agents i and j. We assume throughout that links are *undirected*, in the sense that we do not specify whether link ij points from i to j or vice versa. Let $N_i(\mathbf{g}) = \{j : ij \in \mathbf{g}\}$ and let $Q(\mathbf{g}) = \{i | \exists j \text{ s.t. } ij \in \mathbf{g}\}$.

We say that agents i and j are *connected*, denoted $i \overset{\mathbf{g}}{\leftrightarrow} j$, if there exist $j_1, j_2, ..., j_n$ for some n such that $ij_1, j_1j_2, ..., j_nj \in \mathbf{g}$. In the remainder of this paper, we will only discuss *connected networks*, i.e. networks in which every two agents are connected. A network $\mathbf{g}$ is *complete* if for all i, j such that $i \overset{\mathbf{g}}{\leftrightarrow} j$, we have $ij \in \mathbf{g}$. For agent i, let $M_i(\mathbf{g})$ denote the set of complete sub-networks of $\mathbf{g}$ that involve i: $M_i(\mathbf{g}) = \{\mathbf{g}' \subset \mathbf{g} : \mathbf{g}' \text{ is complete and } i \in Q(\mathbf{g}')\}$. Let $\bar{M}_i(\mathbf{g})$ denote the set of maximal complete sub-networks of $\mathbf{g}$ that involves i. Let $\mathcal{M}_i$ denote the set of all possible complete networks that involve i.

3.2 The Game

Time is discrete and infinite: $t = 1, 2, \cdots$. In every period t, the agents play the following two-stage game.

Stage 1: network formation. The agents simultaneously decide whether to make a costly effort to link with one another, and links are formed or maintained by mutual consent. Formally, each agent i chooses $l_i^t = \{l_{ij}^t\}_{j \neq i}$ where $l_{ij}^t \in \{0, 1\}$, and a link is formed or maintained between i and j if and only if $l_{ij}^t = l_{ji}^t = 1$. l_{ij}^t costs $c \geq 0$ for agent i if link ij is formed or maintained, and incurs no cost otherwise. Given a profile of linking decisions $l^t = \{l_i^t\}_{i \in I}$, let $\mathbf{g}(l^t)$ denote the network that is formed, and let $\phi^t := \{\mathbf{g}(l^t)\}_{\tau=1}^t$ denote a *formation history* or a *formation path* up to time t, with the initial condition that $\phi^0 = (\varnothing, \varnothing)$. Let $\Phi = \{\phi^t : t \in \mathbb{N}\}$ denote the set of all possible formation histories.

At the end of stage 1, the agents observe a *public signal* which is generated by a signal device $y : \Phi \to Y$, where Y is the set of signal realizations. We assume that y and Y are common knowledge. We sometimes refer to y as the *monitoring structure* in the remainder of this paper.

Stage 2: interaction. For network $\mathbf{g}(l^t)$ formed in the network formation stage, a (potentially different) game is played on every non-empty complete sub-network $\mathbf{g}' \subset \mathbf{g}(l^t)$, denoted $\Gamma(\mathbf{g}')$. The players of $\Gamma(\mathbf{g}')$ are the agents in $Q(\mathbf{g}')$. For every $i \in Q(\mathbf{g}')$, let $A_i(\mathbf{g}')$ denote the set of available actions for i in $\Gamma(\mathbf{g}')$, and let $a_i(\mathbf{g}')$ denote an arbitrary action. The available action set of agent i for the entire stage 2 is denoted $\mathcal{A}_i = \times_{\mathbf{g}' \in \mathcal{M}_i} A_i(\mathbf{g}')$. We assume that $\mathcal{A}_i$ is finite.

At the beginning of stage 2, agent i knows $M_i(\mathbf{g}(l^t))$. At the end of stage 2, agent i knows every action of every agent in every game she played in stage 2. Let $\kappa_i^t \subset \{a_j^t : j \in I, \mathbf{g}' \textit{ is complete}, ij \in \mathbf{g}', a_j^t \in A_j(\mathbf{g}')\}$ denote i's knowledge on action history in period

t (note that κ_i^t is sufficient information for $M_i(\mathbf{g}(l^t))$), and let $\mathcal{K} = \{\{\kappa_i^\tau\}_{\tau=0}^t : t \in \mathbb{N}\}$ denote the set of all possible histories of such knowledge.

It is worth noting here that we have different constructions for monitoring the network formation history and the action history. Network formation is *publicly* monitored, but the precision of monitoring varies from full information (y is a one-to-one mapping) to no information (y is a fixed constant). A practical interpretation of this monitoring technology in social networks is news media. On the other hand, actions are monitored *privately*, so an agent has absolutely no information on actions taken outside the games she has played. This assumption is realistic in most applications. Last but not least, monitoring on actions also reveals some local information on network formation, since what games an agent plays in a given period is determined by the current network.

Now we describe the payoffs. In $\Gamma(\mathbf{g}')$, given an action profile $a(\mathbf{g}') = \{a_i(\mathbf{g}')\}_{i\in Q(\mathbf{g}')}$, i's payoff from this game is given by a function $u_i(a(\mathbf{g}')) : \times_{i\in Q(\mathbf{g}')}\mathcal{A}_i \to \mathbb{R}$. We make the following assumption on payoffs.

Assumption 1. *In any game $\Gamma(\mathbf{g}')$, each agent $i \in Q(\mathbf{g}')$ has a strictly dominant action $a_i^D(\mathbf{g}')$.*

This assumption holds for many games studied in both theoretical and empirical literature, such as prisoners' dilemma, gift-giving game and public good provision. As will be seen in the subsequent analysis, it is useful in constructing a straightforward equilibrium strategy profile that sustains a prescribed network and associated actions, under the potentially large and complex set of possible (observable) histories. We denote the action profile for $\Gamma(\mathbf{g}')$ in which every agent plays $a_i^D(\mathbf{g}')$ as $a^D(\mathbf{g}')$.

Let $a_i^t = \{a_i^t(\mathbf{g}')\}_{\mathbf{g}'\in M_i(\mathbf{g}(l^t))}$ denote i's action in the games she plays in stage 2 of period t. Given a sequence of linking efforts $\bar{l} = \{l^t\}_{t\in\mathbb{N}^+}$ which generates a sequence of networks $\{\mathbf{g}(\bar{l}^t)\}_{t\in\mathbb{N}^+}$, and a sequence of actions $\bar{a} = \{\bar{a}_i\}_{i\in I}$ where $\bar{a}_i = \{a^t\}_{t\in\mathbb{N}^+} = \{a_i^t\}_{t\in\mathbb{N}^+}$, we let $U_i(l^t, a^t)$ denote i's payoff in period t. i's total payoff is

$$V_i(\bar{l}, \bar{a}) = \sum_{t=1}^{\infty} \gamma^{t-1} U_i(l^t, a^t) = \sum_{t=1}^{\infty} \gamma^{t-1} \Big(\sum_{\mathbf{g}'\in M_i(\mathbf{g}(l^t))} u_i(a^t(\mathbf{g}')) - \sum_{j:ij\in\mathbf{g}(l^t)} c \Big).$$

In other words, agent i's *one-period payoff* is the sum of payoffs from all the games she plays less her link formation/maintenance cost, and her *discounted total payoff* is the discounted sum of her infinite one-period payoff stream.

3.3 Strategy, Equilibrium and Convergence

Since an agent's strategic decision consists of two parts – forming links and taking actions – and she receives additional local information about the network topology in between, her strategy needs to be defined separately for each part. Let $z_i^t = \{y^\tau, \kappa_i^\tau\}_{\tau=0}^{t-1}$ denote a history consisting of both the realized public signals and the actions that i has observed, and let $\mathcal{Z}$ denote the set of all possible histories. A (pure) strategy σ_i for agent i consists of two mappings $\sigma_i^1 : \mathcal{Z} \to \{0,1\}^{|I|-1}$ and $\sigma_i^2 : \mathcal{Z} \times Y \times 2^{\mathcal{M}_i} \to \mathcal{A}_i$. The first mapping selects a subset of agents that i agrees to link with according to her observed history before the current period, and the second selects an action profile based on the same history *and* new observed information in the current period.

Our equilibrium notion below is a standard one in a history-perfect sense. The slight divergence from the literature on repeated games is that it is neither a pure public equilibrium nor a pure private one, since agents are informed differently about the history of different stages.

Definition 1 (Equilibrium)**.** *A (pure strategy)* ***equilibrium*** *is a vector of strategies* $\sigma^* = (\sigma_1^*, \cdots, \sigma_N^*)$ *such that: for each agent i, each stage of every period t and every possible history of public signal and private knowledge, σ_i^* maximizes agent i's discounted total payoff at period t given σ_{-i}^*.*

We note the existence of an equilibrium below.

Proposition 1. *There exists a pure strategy equilibrium.*

Proof. Every agent i always choosing $l_{ij} = 0$ for every $j \neq i$ in stage 1 and playing $a_i^D(\mathbf{g}')$ for every complete sub-network $\mathbf{g}'$ in stage 2 is an equilibrium by definition. □

In this paper, we will focus on equilibria that lead to a persistent network topology and action profile. We would like to understand what characterizes a network that can remain stable over time, and what behavior can or cannot occur constantly on such a network. This property is denoted as *convergence* of the dynamic process. Fix a network $\mathbf{g}$ and a one-period action profile $a = \{a_i(\mathbf{g}')\}_{\mathbf{g}' \in M_i(\mathbf{g}), i \in I}$, and we have the following definition:

Definition 2 (Convergence)**.** *We say that the dynamic process* ***converges*** *to $(\mathbf{g}, a)$ in equilibrium σ^* if there exists $T \in \mathbb{N}^+$ such that following* ***every*** *(finite) history of network formation and action, according to σ^* the network stays at $\mathbf{g}$ and the action profile stays at a forever after at most T periods.*

4 Results

In this section, we present our main results. We start by constructing a straightforward strategy profile and proving Convergence Theorems that characterize the set of networks and associated action profiles that can be sustained with this strategy profile. Next, we turn to welfare analysis and compare our model with one where network formation is exogenous. Towards the end of this section we provide several representative examples illustrating the relation between sustainable networks and sustainable actions.

4.1 Convergence Theorem with Informative Monitoring Structure

4.1.1 Informative Monitoring Structures

We explicitly construct a strategy profile that serves as a candidate for an equilibrium in the Convergence Theorem established later. Since a strategy involves decision on links and actions based on public signals, we first need to specify the associated monitoring structure that is "sufficiently informative" for convergence.

Fix a network $\mathbf{g}$ and integer $K \geq 1$. We characterize a particular monitoring structure $y_{\mathbf{g},K}$ recursively as follows.

1. $Y = \{C, P\}$, where C represents the *cooperation phase* and P represents the *punishment phase*.

2. $y^0_{\mathbf{g},K} = C$.

3. In period $t \geq 1$: if $y^{t-1}_{\mathbf{g},K} = C$, we distinguish 2 cases:

   – case 1: for every pair of agents $ij \in \mathbf{g}$, $a_{ij} = a_{ji} = 1$ and for every pair of agents $ij \notin \mathbf{g}$, $a_{ij} = 0$ or $a_{ji} = 0$ (or both).

   – case 2: otherwise (i.e. case 1 fails for some pair of agents ij)

   – In case 1, we define $\hat{y}^t_{\mathbf{g},K} = C$ and in case 2, we define $y^t_{\mathbf{g},K} = P$.

4. In period $t \geq 1$, if $y^{t-1}_{\mathbf{g},K} = P$: we again distinguish 2 cases:

   – case 1: $y^{t-2}_{\mathbf{g},K} = y^{t-3}_{\mathbf{g},K} = \cdots = y^{t-K}_{\mathbf{g},K} = P$

   – case 2: otherwise

   – In case 1, we define $y^t_{\mathbf{g},K} = C$ and in case 2, $y^t_{\mathbf{g},K} = P$.

As we will see in the proof of Theorem 1 below access to the information provided by $y_{\mathbf{g},K}$ allows the agents to divide the formation process into two phases: the cooperation phase which continues forever if agents choose their actions in order to form or maintain the network $\mathbf{g}$, and the punishment phase that starts when agents depart from the cooperation phase and continues for K periods. From the public signal C or P, each agent knows what phase she should currently be in, but not how long that phase has lasted or how many times the same phase has occurred before.

The monitoring structure that is applicable to our Convergence Theorem can be generalized from $y_{\mathbf{g},K}$ to a class with at least the same level of informativeness. Consider any other signal structure $\hat{y}$ with signal space $\hat{Y}$. $\hat{y}$ is as informative as $y_{\mathbf{g},K}$ if there is a mapping $\eta : \hat{Y} \to Y$ such that $y_{\mathbf{g},K}(\phi^t) = \eta(\hat{y}(\phi^t))$. That is, $\hat{y}$ reveals at least as much about the history as $y_{\mathbf{g},K}$ (and perhaps more). Notice that complete information about network formation is always as informative as $y_{\mathbf{g},K}$, no matter what $\mathbf{g}$ and K are.

4.1.2 Construction of Equilibrium Strategies

Assume that the monitoring structure is as informative as $y_{\mathbf{g},K}$, and fix a network and a one-period action profile $(\mathbf{g}, a)$. First we define some concepts that are useful for characterizing the equilibrium strategies. In an arbitrary period t, we say that there is a **deviation in network** if the network formed after stage 1 is different from $\mathbf{g}$, and that there is a **deviation in action** if there is some agent j who plays an action different from $a_i(\mathbf{g}')$ in some complete sub-network $\mathbf{g}' \subset \mathbf{g}$.

Consider the following strategy profile, denoted $\sigma(\mathbf{g}, a)$:

For every agent i:

1. In stage 1: start with the *cooperation phase* in period 1: choose $l_{ij}^1 = 1$ if and only if $ij \in \mathbf{g}$. In $t > 1$, if $y^\tau = C$ and $\kappa_i^\tau = \{a_j^\tau : a_j^\tau = a_j(\mathbf{g}'), \mathbf{g}' \in M_i(\mathbf{g})\}$ for all $\tau < t$, then continue the cooperation phase.

2. Otherwise, i enters the *punishment phase*. Consider the first period t when i has observed some agent deviating from the cooperation phase. If the deviation includes a deviation in network (as signaled by $y^{t-1} = P$), then choose $l_{ij}^t = 0$ for every $j \neq i$.

   If the deviation is only a deviation in action, then choose $l_{ij}^t = 0$ for every $j \neq i$ who has also observed the deviation. The identities of such j's can be inferred from κ_i^{t-1}, and we denote the set of these agents as J. For agents in $(I \setminus J) - i$, choose i's one-period *best response* in linking decisions, assuming that those agents and i will play their dominant strategy in every complete sub-network $\mathbf{g}'$ in stage 2. Note that the set of complete sub-networks involving i at the beginning of stage 2 can be determined to be $M_i(\mathbf{g} \setminus \{ij : j \in J\})$, and thus this best response must exist. Then from period $t+1$ onwards, choose $l_{ij} = 0$ for every $j \neq i$. The punishment phase ends in the first period t' after t when $y^{t'-1} = C$, and starting from period t' agent i resumes the cooperation phase, and so on.

3. In stage 2: for every complete sub-network $\mathbf{g}'$, play $a_i(\mathbf{g}')$ if i is in the cooperation phase and $y^t = C$, and play $a_i^D(\mathbf{g}')$ otherwise.

$\sigma(\mathbf{g}, a)$ can be interpreted as the following pattern of behavior: the agents start by cooperating towards building a designated network and conforming to a designated action profile. They form or maintain a link if and only if that link belongs to the specific network $\mathbf{g}$, and they take the specific action in a when playing each game on $\mathbf{g}$. Once a deviation is detected by an agent – no matter whether it is a deviation in link or a deviation in action – the agent first severs her link with the deviator, and then changes her behavior to playing a myopic best response. She resumes cooperation after K periods, when the network topology is publicly known to return to $\mathbf{g}$.

4.1.3 Convergence Theorem

We begin with a simple observation: for the dynamic process to converge in equilibrium, each agent must receive a non-negative payoff in the long run.

Proposition 2. *If there exists an equilibrium σ^* where the dynamic process converges to $(\mathbf{g}, a)$, then for every $i \in I$, $\sum_{\mathbf{g}' \in M_i(\mathbf{g})} u_i(a(\mathbf{g}')) - \sum_{j:ij \in \mathbf{g}} c \geq 0$.*

Proof. Suppose that there exists an equilibrium where the dynamic process converges to $(\mathbf{g}, a)$, and that $\sum_{\mathbf{g}' \in M_i(\mathbf{g})} u_i(a(\mathbf{g}')) - \sum_{j:ij \in \mathbf{g}} c < 0$ for some i. Then on the equilibrium path when $\mathbf{g}$ has been formed and will persist forever, i is always strictly better off by deviating to a strategy in which $l_{ij} = 0$ always and obtaining payoff 0 thereafter. This is a contradiction to the assumption of an equilibrium. □

Our first Convergence Theorem is a partial converse to above: if the inequality is strict for all agents, the monitoring structure is sufficiently informative (in particular if the monitoring structure yields complete information) and agents are sufficiently patient then our constructed strategy profile is an equilibrium in which the formation process converges to $(\mathbf{g}, a)$.

Theorem 1. *Suppose that $(\mathbf{g}, a)$ is such that $\sum_{\mathbf{g}' \in M_i(\mathbf{g})} u_i(a(\mathbf{g}')) - \sum_{j:ij \in \mathbf{g}} c > 0$ for every $i \in I$. Then there exists integer K and a cutoff $\bar{\gamma} \in (0, 1)$ such that when $\gamma \in [\bar{\gamma}, 1)$ and the monitoring structure is as informative as $y_{\mathbf{g},K}$, $\sigma(\mathbf{g}, a)$ is an equilibrium where the dynamic process converges to $(\mathbf{g}, a)$.*

Proof. We prove the result by considering a one-step deviation by every agent i, for every possible history of public signal and action that can be observed by i. We start with stage 2 in an arbitrary period t, and distinguish two cases.

Case 1: i is in the punishment phase or $y^t = P$. According to $(\mathbf{g}, a)$, agent i's action in period t does not affect any agent's decision in the subsequent periods. Hence, playing $a_i^D(\mathbf{g'})$ in every complete sub-network $\mathbf{g'}$ is a best response.

Case 2: i is in the cooperation phase and $y^t = C$. Let $\bar{v} \geq 0$ denote the largest one-period gain that i has in any game by any one-step deviation from any action profile. Since the number of games and the number of each agent's actions in every game are both finite, we know that $\bar{v}$ exists. Suppose that i deviates, the difference in her payoff from no deviation is bounded above by

$$\bar{v}|M_i(\mathbf{g})| + \gamma\bar{v}|M_i(\mathbf{g})| - \sum_{\tau=2}^{K+1} \gamma^\tau \Big(\sum_{\mathbf{g'} \in M_i(\mathbf{g})} u_i(a(\mathbf{g'})) - \sum_{j:ij\in\mathbf{g}} c \Big).$$

By assumption, $\sum_{\mathbf{g'}\in M_i(\mathbf{g})} u_i(a(\mathbf{g'})) - \sum_{j:ij\in\mathbf{g}} c > 0$. Then we can find integer K_1 such that $K_1(\sum_{\mathbf{g'}\in M_i(\mathbf{g})} u_i(a(\mathbf{g'})) - \sum_{j:ij\in\mathbf{g}} c) > 2\bar{v}|M_i(\mathbf{g})|$. When $K = K_1$ and γ is sufficiently close to 1, no deviation is i's best response.

Now we consider stage 1. Again we distinguish two cases.

Case 1: i is in the punishment phase. According to $(\mathbf{g}, a)$, every other agent is choosing 0, so i's linking decision does not make a difference. Trivially this is a best response.

Case 2: i is in the cooperation phase. Let $\hat{v} \geq 0$ denote the largest one-period gain that i has in this stage by any one-step deviation from any strategy profile. Since the number of networks, the number of games and the number of each agent's actions in every game are all finite, we know that $\hat{v}$ exists. Suppose that i deviates, the difference in her payoff from no deviation is bounded above by

$$\hat{v} - \sum_{\tau=1}^{K} \gamma^\tau \Big(\sum_{\mathbf{g'} \in M_i(\mathbf{g})} u_i(a(\mathbf{g'})) - \sum_{j:ij\in\mathbf{g}} c \Big).$$

Similar to above, we can find integer K_2 such that $K_2(\sum_{\mathbf{g'}\in M_i(\mathbf{g})} u_i(a(\mathbf{g'})) - \sum_{j:ij\in\mathbf{g}} c) > \hat{v}$. When $K = K_2$ and γ is sufficiently close to 1, no deviation is i's best response. Then we can let $K = \max\{K_1, K_2\}$ to obtain our desired K such that $\sigma(\mathbf{g}, a)$ is an equilibrium.

Finally, convergence follows from its definition and the characterization of $\sigma(\mathbf{g}, a, L)$. This completes the proof. □

This result asserts that when the monitoring structure is sufficiently informative and agents are sufficiently patient, the simple strategy profile we propose is an equilibrium with convergence. The range of network and action profile that can be converged to is rather broad: it contains most such network-action combination that the dynamic process can possibly converge to, except the special boundary cases with some agents having a zero payoff. It is useful to contrast this result with the familiar Folk Theorem in repeated games: the Folk Theorem focuses on *payoffs* that can be achieved in the long run, possibly by unstable and random equilibrium behavior; the Convergence Theorem focuses on *networks and action profiles* that can persist in the long run. We aim at identifying structures of connections and behavior that are stable over time.

It is also worth mentioning how our prescribed strategy constitutes best responses under all possible public and private observed histories, which can be extremely complex in principle. Two factors are crucial: unilateral link severance and informative monitoring on network formation. Even if an initial deviation is only privately observed, it will become "public" in the next period because the corresponding link(s) will be severed, which will then be reflected in the public signal. The short time interval (one period) before a private deviation is revealed in the form of link severance to the public ensures that using myopic best response is optimal in the period when the deviation occurs. Once punishment is announced by the public signal, the one-period Nash equilibrium that no agent agrees to form any link serves as a punishment profile that yields each agent her minmax payoff 0. Hence, cooperation can be incentivized as long as the punishment phase is sufficiently long, but not necessarily forever so that the process can still return to cooperation after any history. In the next section, we present a different Convergence Theorem that states the consequence of taking out the monitoring structure.

4.2 Convergence Theorem without Monitoring Structure

In this section we consider the environment without the public signal device y. A strategy for an agent then consists of mappings from her private knowledge only, to her available decision sets in each stage. Specifically, it consists of two mappings $\sigma_i^1 : \mathcal{K} \to \{0,1\}^{|I|-1}$ and $\sigma_i^2 : \mathcal{K} \times 2^{\mathcal{M}_i} \to \mathcal{A}_i$.

4.2.1 Construction of Equilibrium Strategies

As before, we explicitly construct a strategy profile that will be used for our Convergence Theorem. Fix a network and a one-period action profile $(\mathbf{g}, a)$, and consider the following strategy profile, denoted $\sigma(\mathbf{g}, a)$:

For every agent i, consider every maximal complete sub-network $\tilde{\mathbf{g}} \in \bar{M}_i(\mathbf{g})$:

1. In stage 1: start with the *cooperation phase* (for $\tilde{\mathbf{g}}$) in period 1: choose $l_{ij}^1 = 1$ if and only if $ij \in \tilde{\mathbf{g}}$. In $t > 1$, if $\tilde{\mathbf{g}} \in \bar{M}_i(\mathbf{g}(l^\tau))$ and $\kappa_i^\tau = \{a_j^\tau : a_j^\tau = a_j(\mathbf{g}'), \mathbf{g}' \in M_i(\mathbf{g})\}$ for all $\tau < t$, then continue the cooperation phase.

2. Otherwise, i enters in the *punishment phase*. Consider the first period t when i has observed some agent deviating from the cooperation phase. If the deviation includes a deviation in network (as signaled by $\tilde{\mathbf{g}} \notin \bar{M}_i(\mathbf{g}(l^{t-1}))$), then choose $l^t_{ij} = 0$ for every $j \neq i$.

   If the deviation is only a deviation in action, then choose $l^t_{ij} = 0$ for every $j \neq i$ who has also observed the deviation. The identities of such j's in $\tilde{\mathbf{g}}$ can be inferred from κ_i^{t-1}, and we denote the set of these agents as J. For agents in $(Q(\tilde{\mathbf{g}}) \setminus J) - i$, choose i's one-period *best response* in linking decisions in $\tilde{\mathbf{g}}$, assuming that those agents and i will play their dominant strategy in every game within $\tilde{\mathbf{g}}$ in stage 2. Note that the set of complete sub-networks of $\tilde{\mathbf{g}}$ involving i at the beginning of stage 2 can be determined to be $M_i(\tilde{\mathbf{g}} \setminus \{ij : j \in J\})$, and thus this best response must exist. Then from period $t+1$ onwards, choose $l_{ij} = 0$ for every $j \neq i$. The punishment phase ends after K periods after the first period t' when $\tilde{\mathbf{g}} \notin \bar{M}_i(\mathbf{g}(l^{t'}))$. After that, i resumes the cooperation phase, and so on.

3. In stage 2: in every period t, for every complete sub-network $\mathbf{g}' \subset \tilde{\mathbf{g}}$, play $a_i(\mathbf{g}')$ if i is in the cooperation phase and $\tilde{\mathbf{g}} \in \bar{M}_i(\mathbf{g}(l^t))$, and play $a_i^D(\mathbf{g}')$ otherwise.

The behavior described by this strategy profile can be understood as enforcing cooperation *locally* with a "grim trigger" strategy on every maximal complete sub-network. Whenever a deviation is observed, the agents involved begin punishment by choosing their myopic best response first and then severing all links *within the corresponding maximal complete sub-network*, but they remain cooperative in other maximal complete sub-networks.

4.2.2 Convergence Theorem

We present the Convergence Theorem without monitoring structure below.

Theorem 2. *Suppose that $(\mathbf{g}, a)$ satisfies the following conditions:*

1. $\sum_{\mathbf{g}' \in M_i(\tilde{\mathbf{g}})} u_i(a(\mathbf{g}')) - \sum_{j:ij \in \tilde{\mathbf{g}}} c > 0$ for every $i \in I$ and every $\tilde{\mathbf{g}} \in \bar{M}_i(\mathbf{g})$.

2. For every $i \in I$ and every two maximal sub-networks $\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_2 \in \bar{M}_i(\mathbf{g})$, $\tilde{\mathbf{g}}_1 \cap \tilde{\mathbf{g}}_2 = \varnothing$.

Then there exists integer K and a cutoff $\bar{\gamma} \in (0,1)$ such that when $\gamma \in [\bar{\gamma}, 1)$ and the monitoring structure is as informative as $y_{\mathbf{g},K}$, $\sigma(\mathbf{g}, a)$ is an equilibrium where the dynamic process converges to $(\mathbf{g}, a)$.

Proof. Apply the argument in the proof of Theorem 1 to every maximal complete sub-network $\tilde{\mathbf{g}} \in \bar{M}_i(\mathbf{g})$ for every i. $\square$

This result illustrates that even though global cooperation cannot be enforced due to the absence of public signals, the agents can still sustain local cooperation. Here the network topology itself serves as a locally public signal: since every agent can observe the complete sub-networks they are in, the topology of every maximal complete sub-network is common knowledge among its members. Hence, when such a sub-network provides each member with a positive payoff and no two such sub-networks overlap, the argument for Theorem 1 can be applied.

To contrast with Theorem 1, we demonstrate with examples how cooperation fails if either of the two conditions in Theorem 2 does not hold. First, consider a group of 5 agents and the following network: $\{12, 23, 13, 14, 15, 45\}$ (Figure 1(A)). Agent 1 is a "pivotal" agent in this network, in the sense that she is the only agent that are involved in both triangles $\{12, 23, 13\}$ and $\{14, 15, 45\}$. As a result, if the sub-network $\{12, 23, 13\}$ yields a negative payoff for agent 1 (violating condition 1 above), she will never be willing to remain linked with agents 2 and 3 even if her other maximal complete sub-network $\{14, 15, 45\}$ can compensate her with a large positive payoff. She could simply sever her links $12, 13$ and stick to her original links and actions in the other triangle, because without a public signal agents 4 and 5 will never realize any deviation among agents 1, 2 and 3.

Next, we discuss a violation of the second condition. Consider a group of 4 agents and the following network: $\{12, 23, 13, 14, 24\}$ (Figure 1(B)). Suppose that a deviation occurs in the game $\Gamma(\{23\})$ in period t. According to the strategy profile $\sigma(\mathbf{g}, a)$, punishment begins in triangle $\{12, 23, 13\}$ in period $t+1$ and link 12 will be severed in period $t+2$. However, period $t+2$ is regarded as the time of an initial deviation in triangle $\{12, 14, 24\}$, so there is time inconsistency for agents 1 and 2 as for when to resume link 12. This timing issue is even more complex if many maximal complete sub-networks overlap, and we will leave studying coordinated punishment with local information for future research.

4.3 Welfare Results

In this section, we present results related to social welfare, i.e. the sum of all agents' payoffs. First we illustrate a simple relation between welfare improvement and equilibrium sustainability.

Definition 3. *For two network-action combinations* $(\mathbf{g}, a)$ *and* $(\hat{\mathbf{g}}, \hat{a})$*, we say that* $(\hat{\mathbf{g}}, \hat{a})$

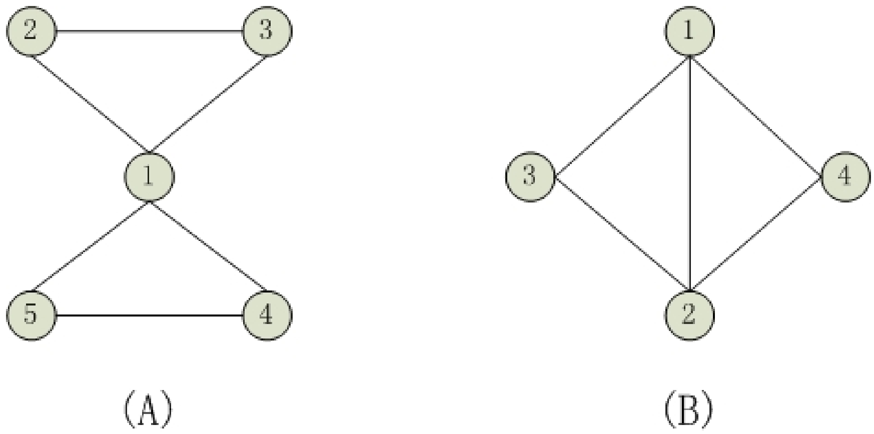

Figure 1: cooperation failure with violation of either condition

is an ***improvement*** *on* $(\mathbf{g}, a)$ *if for every* $i \in I$, $\sum_{\mathbf{g}' \in M_i(\hat{\mathbf{g}})} u_i(\hat{a}(\mathbf{g}')) - \sum_{j: ij \in \hat{\mathbf{g}}} c \geq \sum_{\mathbf{g}' \in M_i(\mathbf{g})} u_i(a(\mathbf{g}')) - \sum_{j: ij \in \mathbf{g}} c$.

An improvement occurs if each agent enjoys a higher one-period payoff. Such improvement may occur on network (e.g. adding/deleting links with otherwise unchanged actions), action (keeping the network topology but choosing different actions) or both. The following result is a corollary of Theorem 1.

Corollary 1. *Fix* $(\mathbf{g}, a)$. *Suppose that* $(\hat{\mathbf{g}}, \hat{a})$ *is an improvement on* $(\mathbf{g}, a)$, *and that* $(\mathbf{g}, a)$ *satisfies the condition in Theorem 1. Then there exists integer* K *and a cutoff* $\bar{\gamma} \in (0, 1)$ *such that when* $\gamma \in [\bar{\gamma}, 1)$ *and the monitoring structure is as informative as* $y_{\mathbf{g},K}$, $\sigma(\mathbf{g}, \hat{a})$ *is an equilibrium in which the dynamic process converges to* $(\mathbf{g}, \hat{a})$.

This result is a positive one on welfare because it shows that every improvement upon the current equilibrium network and action profile, no matter what the current equilibrium prescribes, is feasible once agents are patient enough. The reason for our difference with negative results in the literature (e.g. Watts[34] and Dutta et al.[9]) lies in agent foresight and informative record of history. The literature has shown that the absence of either factor leads to unavoidable inefficient outcomes, while we prove that the combination of both enables welfare improvement.

Next, we compare our model with one under exogenous network formation, in order to evaluate the social consequence of letting agents make linking decisions. We begin with the definition of strong efficiency.

Definition 4. *Given cost* c, *a combination of network and action profile* $(\mathbf{g}(c), a(c))$ *is* ***strongly efficient*** *if it yields the highest possible sum of one-period payoffs.*

Since the strongly efficient $(\mathbf{g}(c), a(c))$ is unique. Note that this is a stronger definition than Pareto efficiency because it requires the maximization of the sum of payoffs. This definition is widely adopted by the literature on network formation and provides a straightforward benchmark of social optimality.

Now we introduce the alternative model to compare with. Consider the following *game with exogenous network*: in stage 1 of every period t, instead of forming a network by the agents' choices, there is a probability distribution G that assigns a network $\mathbf{g}^t$. The rest of the game remains the same as before. Given discount factor γ and cost c, let $W_{EX}(\gamma)$ denote the maximum social welfare (sum of all agents' payoffs) that can be achieved in equilibrium in the game with exogenous network, and let $W_{EN}(\gamma, c)$ denote the maximum social welfare that can be achieved in equilibrium with convergence in the game with endogenous network (the original game).

Theorem 3. *Assume that in* $(\mathbf{g}(0), a(0))$*, every agent gets a positive one-period payoff. Then there exist cutoffs* $\bar{\gamma} \in (0, 1)$ *and* $\bar{c} > 0$ *such that when* $\gamma \in [\bar{\gamma}, 1]$*,* $c \in [0, \bar{c})$ *and the monitoring structure is sufficiently informative,* $W_{EX}(\gamma) < W_{EN}(\gamma, c)$*.*

Proof. According to Theorem 1, when $c = 0$ there exists a cutoff $\bar{\gamma} \in (0, 1)$ such that when $\gamma \in [\bar{\gamma}, 1]$ and the monitoring structure is sufficiently informative, $\sigma(\mathbf{g}(0), a(0))$ is an equilibrium where the dynamic process converges to $(\mathbf{g}(0), a(0))$.

By the definition of $\sigma(\mathbf{g}(0), a(0))$, it yields a higher social welfare than $W_{EX}(\gamma)$. By assumption, in $(\mathbf{g}(0), a(0))$ every agent gets a positive one-period payoff, and thus there exists $\bar{c} > 0$ such that when $c \in [0, \bar{c})$, the social welfare from $(\mathbf{g}(0), a(0))$ is still higher than $W_{EX}(\gamma)$ and $\sigma(\mathbf{g}(0), a(0))$ is still an equilibrium where the dynamic process converges to $(\mathbf{g}(0), a(0))$. Finally, by the definition of $W_{EN}(\gamma, c)$, $W_{EN}(\gamma, c)$ is at least as large as the social welfare from $\sigma(\mathbf{g}(0), a(0))$. Therefore we have $W_{EX}(\gamma) < W_{EN}(\gamma, c)$. $\square$

Theorem 3 provides a condition under which our model with endogenous network formation is superior in welfare to a model with exogenous network formation. This condition is satisfied in many typical network games, e.g. the widely studied prisoners' dilemma. From the perspective of designing an environment for individuals to interact, this result provides a key implication on policy: it is socially desirable to bestow agents the freedom to choose their social or professional circles and lower the cost of establishing relationships, as compared to a predetermined and rigid structure of communication. As long as agents share the norm of what to pursue, social optimum can persist as a self-fulfilling prophecy in equilibrium.

4.4 Examples

In this section, we apply our theoretical results to a particular game to illustrate some interesting implications from the model.

4.4.1 Interrelation between Network and Game

Consider the following environment: $N = 3$, $c = 0.5$. Each possible game $\Gamma(\mathbf{g}')$ is described as follows: suppose that there are n players in the game. Every player has two available actions C (contribute) and D (defect). If all agents choose D, they get a payoff of 1 each. If at least one agent chooses C, whoever chooses C gets 0 each while whoever chooses D gets $n+1$ each. In other words, contribution is purely altruistic. It is clear that D is the strictly dominant action in each game.

First, we demonstrate how the network topology may affect sustainable actions. Consider the network $\mathbf{g} = \{12, 23, 13\}$, and the following action profile a: 1 plays C and $2, 3$ play D in $\Gamma(\{12, 23, 13\})$; 1 plays D and 2 plays C in $\Gamma(\{12\})$; 1 plays D and 3 plays C in $\Gamma(\{13\})$; 2 plays C and 3 plays D in $\Gamma(\{23\})$. This is (one of) the (strongly) efficient action profile given the network. According to Theorem 1, $\sigma(\mathbf{g}, a)$ is an equilibrium where the dynamic process converges to $(\mathbf{g}, a)$ when γ is close to 1. Hence efficient contribution can be sustained in network $\mathbf{g} = \{12, 23, 13\}$.

Now consider the network $\{12, 23\}$. In this network, efficient contribution (one and only one agent contributes in each game) cannot be sustained in this network. To see this, note that for efficient contribution to take place, at least one agent will get a payoff of 0 from her game(s), and thus a strictly negative payoff in total because of the link maintenance cost. Then a strictly better decision for the agent is to sever all her links and get a payoff of 0.

Conversely, changing the nature of games can have a significant impact on sustainable networks. Suppose that the game on $\{12, 23, 13\}$ is replaced by the following one: there is only one action for each agent, and the payoff is -5 for each agent. Now it is possible to sustain an open triangle (e.g. $\{12, 23\}$) with an appropriate action profile (e.g. all agents take action D), but it is not possible to sustain the closed triangle $\{12, 23, 13\}$ with any action profile.

4.4.2 Impossibility of Global Sustainability without Monitoring Structure

Our second example shows the importance of an informative monitoring structure for enforcing global cooperation. Consider the same environment as above, except for that there are now N agents. Consider the wheel network $\{12, 23, \cdots, (N-1)N, 1N\}$, and the following action profile a: 1 plays C and 2 plays D in $\Gamma(\{12\})$; 2 plays C and 3 plays D in $\Gamma(\{23\})$; $\cdots$; N plays C and 1 plays D in $\Gamma(\{1N\})$. This is one of the action profiles that yields the highest possible social welfare conditional on $\mathbf{g}$. It is clear that $(\mathbf{g}, a)$ satisfies the condition in Theorem 1 and hence can be sustained with the presence of an informative monitoring structure. The range of γ for sustaining $(\mathbf{g}, a)$ is not affected by N.

Consider the case without monitoring structure. For agent 1, if she deviates in period t by playing D in $\Gamma(\{12\})$ or simply severs the link 12, it is not going to affect anything in her only other game $\Gamma(\{1N\})$ by period $t + N - 1$ in any equilibrium unless she deviates against agent N by herself. Hence, for every $\gamma \in (0, 1)$, there exists integer N such that agent 1 (as well as every other agent) has an incentive to deviate. This argument can be applied to show the following general result:

Proposition 3. *For every $\gamma \in (0, 1)$, $N(\gamma)$ exists such that when there are at least $N(\gamma)$ agents, if the dynamic process converges to $(\mathbf{g}, \mathbf{a})$ in equilibrium where $\mathbf{g}$ is a wheel network, then every agent must always play D in a.*

This result does not only refer to a wheel network, but represents a general property of a *sparse* network. Without a monitoring structure (or when the monitoring structure is not sufficiently informative) or a sufficiently dense network, an agent will expect her deviation in one game to affect her payoffs in other games only after a long period of time. As a result, it is impossible for her to remain cooperative in the long run. It implies that when public information is limited, patience alone is not enough for sustaining cooperation: agents also need to be connected closely enough to ensure a fast response to every possible deviation.

5 Conclusion

In this paper, we proposed a model that combines network formation and network games. The principal innovation of our paper is the view of networks as having two fundamental functions: an underlying structure for interaction, and a channel of information about other players' behavior. In our framework, both whom to link with and what actions to take in games played on the resulting network are part of an agent's strategic decision. Our results contrast sharply with the literature on pure network formation as well as the literature on games under exogenous network topologies, in that we find non-negligible interdependency between networks and actions in equilibrium. Also, we emphasize on the importance of an informative monitoring structure, and show that the impact of networks is even more significant in the absence of informative public monitoring.

We believe that our framework can be the foundation of several interesting research directions. One possible topic is to consider a specific class of network games and to study the relation between equilibrium behavior and important aspects of equilibrium networks, such as clustering, centrality and proximity. Also, it would be interesting to bring in a communication system (such as a language for cheap talk) into the model, which will enable a more comprehensive analysis of social circles and interpersonal relationships in particular.

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